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Xenakis and chance

Duel and *Stratégie* by Xenakis are musical games involving random or stochastic elements. JANE WAUGH, composer and ethnomusicologist, and W.A.O'N. WAUGH, who specialises in stochastic process theory, examine the rules of these games.

During the past 20 years or so, composers in many countries have been writing works which amount to musical games; This is to say that the players are divided into two or more sides or teams, that they may collaborate in part, but in part they also compete or oppose one another. Thus different performances of the same work will not necessarily sound the same, nor be of the same fixed duration, and in other ways the composer may break with the traditional idea of a musical composition as something he has laid down in a score, leaving only the usual variations of tempo, timbre, and interpretation.

Let us at once say, most emphatically, that the word 'game' is not intended to imply any lack of seriousness in the work. The present article will refer to the mathematician's notion of 'game theory' which, while it encompasses games (in the ordinary sense) like chess and poker, has had its main growth and applications in connection with more serious fields of human conflict like economics, politics, and war. It is the element of conflict, and particularly the mixture of conflict with co-operation, that is fundamental to the theory, and which we shall show to be fundamental to this type of composition. We shall also see that the element of chance or probability is fundamental too.

Although composers in the most modern tradition are thus currently interested in games, these have also appeared in many folk traditions. An excellent example is the Welsh 'Penillion', in which a harpist and a vocalist play a game: the harpist may suddenly change his tune and the vocalist must follow, instantly changing what he is singing, under strict rules and with great elaborations such as internal rhymes in the lines he sings. In classical Indian music a similar situation occurs between the tabla and sarod (or sitar) player.

The balance between co-operation and competition is a subtle one. Competition may be almost completely absent, as when a tennis player practises his strokes against a wall, competing only against the difficulties of the returns he has set up for himself. Here there is obviously not an opposing side, but there is still something to oppose. In Indian music a solo tabla player may play a game of this kind, when he utters rhythmic 'vocables' as rapidly as possible, then immediately proceeds to imitate these sounds on his tabla.

As already mentioned, many composers have 'tried their hand' at game theory. One who, with the advantage of both mathematical training and musical ability, has often combined them successfully is Iannis Xenakis, the Greek-born composer. In this article, we intend to deal only with his use of game theory in the works Duel (1958-59), and in a more complicated, though similarly constructed, work, Stratégie (1962). Both works are written for two orchestras, each with a conductor. The two orchestras compete according to rules described by Xenakis in his book Formalized Music (London 1971).

In this book the composer describes his use of the 'two-person zero-sum game' in these two works, and this term had better be explained in more detail before continuing. A small, but necessary point is that this description may refer to something played by two individuals, or by two teams, using the word 'person' rather like lawyers do when they apply it to a company. The essence is that there are two sides. One can visualise the two conductors more or less competing against one another, each using his own orchestra as his instrument. The term 'zero-sum' refers to the method of scoring. In simple terms it means that one side's gain is the other side's loss, so that if you add up the total gain and loss of both at any time, they will balance out. A game does not have to be 'zero-sum', though many (especially gambling games) are.

In Duel the composer has provided five musical entities which he calls 'events'. These are:

Event I	A cluster of sonic grains such as pizzicati, blows with the wooden part of the bow, and very brief arco sounds distributed stochastically.
Event II	Parallel sustained strings with fluctuations.
Event III	Networks of intertwining string glissandi.
Event IV	Stochastic percussion sounds.
Event V	Stochastic wind instrument sounds.
Event VI	Silence.

The conductors direct with their backs to each other, and an 'exchange' consists of each one choosing an event and directing his section to play it. The complete game consists of a series of exchanges. The 'events' are thus played in pairs, and each pair is evaluated as pleasant, or unpleasant to hear. For example:

I with V is rated very good g^{++}
while
II with III is rated passable p
also
I with I is rated passable p

Note that if one conductor chooses I, the other can produce a wide range of qualities. He can choose so as to produce a combination that might only be p but might be g^{++}. Now we come to the first curious situation. Let us call the conductors X and Y. Then X is to try to choose so as to secure the best combinations, and Y is to oppose him by trying to secure the poorest. This certainly introduces an element of conflict into the situation, but its effect in producing a musical performance which is pleasing, or otherwise, is obscure. Conflict, however, is required if the performance is to be analysed according to the mathematical theory of games. Anyway, here is the complete table of evaluations:

(X) \ (Y)	I	II	III	IV	V	VI
I	p	g	g^{++}	g^{+}	g^{+}	p
II	g	p	p	g	p^{+}	p
III	g^{++}	p	p	g^{+}	g	p
IV	g^{+}	g	g^{+}	p	g	p
V	g^{+}	p^{+}	g	g	p	p
VI	p	p	p	p	p	p^{-}

Now suppose X's choice is down the side and Y's is along the top. Obviously Y can do pretty well: all he needs to do is choose column VI all the time and whatever X does the combination will either be passable (p) or worse (p^{-}).

So far the game looks a pretty poor one. So the mathematical manipulations begin, using game theory. Here we want to spare you the details, but we will have to try to give you some idea of what is going on. Firstly, mathematical game theory operates with numerical 'payoffs' rather than qualitative ones. Xenakis therefore quantifies according to what he describes as 'a rough numerical scale':

p^{-}	p	p^{+}	g	g^{+}	g^{++}
0	1	2	3	4	5

It is worth noting that if he used a different scale everything that follows could turn out differently. For example, he could have given much greater weight to the preferable qualities with the scale:

p^-	p	p^+	g	g^+	g^{++}
0	1	2	4	8	16

Why choose one rather than the other? We don't know and he doesn't say.

The numerical form of the evaluations is the following square array of numbers, called a 'matrix':

		(Y)					
		I	II	III	IV	V	VI
	I	1	3	5	4	4	1
	II	3	1	1	3	2	1
	III	5	1	1	4	3	1
(X)	IV	4	3	4	1	3	1
	V	4	2	3	3	1	1
	VI	1	1	1	1	1	0

The gains and losses are now worked out according to this matrix, at each 'exchange'. For example, if X chooses III (look down the side) and Y chooses I (look along the top) we see that in the row and column chosen the figure is 5. This figure is the amount Y must pay to X. The game is a zero-sum one because the total of X's gain (plus 5) and Y's loss (minus 5) is zero, and so on for any other pair of choices.

Let us abbreviate the choice just described as (III, I); then (I, III) will mean that X chooses I and Y chooses III. Note that on (I, III) Y must still pay 5 to X which is a kind of symmetry, and this game is at present symmetric throughout (check, for example, the score 3 on both (IV, II) and (II, IV)).

Xenakis now proceeds in stages to modify the matrix to produce a game that possesses further game-theoretic properties. Some of the modifications look pretty arbitrary. For example, his very first step is, without comment, to modify the score on (VI, VI) from 0 (which is p^-) to 3 (which is g). Note that this pair is the peculiar one silence v. silence, and it seems at least likely that a change in your evaluation of that is going to change the game considerably.

The game at the moment looks pretty poor for Y, because all 'payoffs' (understood as from Y to X) are positive or zero. Thus X can never lose and Y can never gain. You might say this serves Y right, since his rôle is to try to get the least pleasant sounds played, but this is not the idea, and at a later stage Xenakis levels things up by making some 'payoffs' negative (X pays Y). However, a preliminary step is to break the symmetry. For example, whereas, before, the payoff on (IV, II) and (II, IV) was 3, he takes (IV, II) as 4, keeping (II, IV) as 3. He also again quietly jacks up the payoff on (VI, VI) or silence v. silence from 3 to 4, but he doesn't say anything about that. He prints a whole sequence of matrices embodying these and subsequent modifications, but we will not reproduce them all, since they can be found in <u>Formalized Music</u>.

His next step brings in probabilities. Here we will try to keep things elementary, but we must ask you to hold on to your hats and try to stay with us.

Some games are rather dull, in that there is a 'best' thing (called 'strategy') that each player can adopt each time they play the game. Look back to the matrix and you will see that Y might as well play VI (silence) every time. Then X can play anything from I to V and the payoff will be 1 (=p) while if he plays VI then the payoff is zero (=p^-) which is even better for Y.

Other games are better balanced between the two players. Manipulations which we will omit lead Xenakis to a new payoff matrix. It is delivered from the old one but, as we said above, it allows both positive payoffs (Y pays X) and negative ones (X pays Y), which makes sense. Other adjustments have been made in ways which are consistent with the requirements of game theory. The result is shown on the opposite page.

Now the situation has radically changed. An obviously good column for Y to play is VI (still silence) because he might win 3 from X if Y chose III, and would win 1 if X chose anything but VI. But if X knows Y has chosen VI so will X, <u>and he will win 3 from Y</u>.

The point is that if one player knows, or can even guess, the other's strategy, he can win. It is pretty obvious that what both of them should do is to keep switching all the time to baffle the other. X should choose rows which are good for him often, but not too often or Y will rumble him, so he must mix in all the choices. Similarly for Y.

What is less obvious, but is one of the most fundamental and most interesting

(Y)

(X)	I	II	III	IV	V	VI
I	-1	+1	+3	-1	+1	-1
II	+1	-1	-1	-1	+1	-1
III	+3	-1	-3	+5	+1	-3
IV	-1	+3	+3	-1	-1	-1
V	+1	-1	+1	+1	-1	-1
VI	-1	-1	-3	-1	-1	+3

theorems of game theory, is that there is a best or 'optimum' proportion in which X should play the various strategies I to VI, and also an optimum (though in different proportions) for Y. To avoid having his opponent detect his pattern of play (in which case the opponent could improve his chances), each one should play his choices in a random sequence, but working it so as to balance out in the long run to his optimum proportions (called his 'optimum mixed strategy' if you ever want to look up some more game theory). The proportions for Xenakis' game are:

Strategy	I	II	III	IV	V	VI	
X	14	6	6	6	8	16	(all out of 56)
Y	19	7	6	1	7	16	

Another point, which we have not mentioned, that helps to guide Xenakis' manipulations is that he tries to make his game 'fair'. This is not the same as zero-sum. After all, you could lay 2 to 1 (your gain is your opponent's loss) on an even money chance like coin tossing, but you would be rather foolish to do so. At each exchange your gain is your opponent's loss and vice versa, but on average he wins 2 half the time and only loses 1 for the other half, so on the whole he will gain steadily. Mainly because he tries to keep his payoffs as simple small whole numbers, which do not allow very fine adjustments, Xenakis ends up with a game that, on average and in the long term, gives Y about a 7% advantage - rather better than the advantage of 2 in 35 that is taken at roulette, traditionally, by casinos.

So Xenakis stops there, and proceeds to the analysis of Stratégie.

<u>Stratégie</u> is a similar game but much more complex: each conductor has a choice of 19 events instead of 6 (they are made up of 6 basic events and some compounds of these). His analysis is much briefer and includes proposals for simplifying the scoring system so that all of the 19 x 19 = 361 choices need not be examined individually. Otherwise it is basically the same. For both games, Xenakis also provides a few suggestions of a purely organisational nature, such as the provision of referees or scorers, prior decision by the conductors to play for a fixed number of minutes or a fixed number of engagements, and so on.

So what are we to conclude from this? Xenakis has certainly achieved one thing: he has written two works with a clearly defined plan to produce substantial differences in what the audience hears each time they are performed. If you consider (and we do) that music which essentially varies from one performance to another has an attraction, like a mobile sculpture, then this is good.

On the other hand, if we have managed to make our analysis at least partly clear, we are sure you will recognise that there is a strongly mechanistic element in the successive modifications he makes to his 'payoff matrix'. He commits himself to follow a set of rules which lead him to a game which has certain qualities (zero-sum, fairness) which belong to game theory. He says almost nothing about whether he thinks these game-theoretic qualities correspond to aesthetically pleasing qualities in an actual performance. Perhaps he does not care. Almost the sole exception is a remark on page 118 of <u>Formalized Music</u>:

"The sonic processes derived from the two experiments are, moreover, satisfactory". We think this could be correctly translated as: "It sounded quite good when we played it".

In the present day climate of opinion many people may not find this particularly significant. What may disturb some people more is that we seem to be approaching what is sometimes referred to (pejoratively) as 'machine music'. We have emphasised the way in which Xenakis develops these two works by adopting a strict set of rules (those of game theory) and apparently letting them lead where they will. We did so deliberately, because this is a central portion of the picture. But is not all of it, and we must redress the balance.

Firstly, like most systems of law, the rules allow more initiative to someone who is master of them than might appear at first glance. Secondly, we both believe that he is doing something very important by bringing into one of the arts ideas which are central to present-day scientific thinking, and are constantly spreading into other fields. We are moving into a period when we must think in terms of fluidity, variable performance and probability, and the tools for dealing with these include the theories of games and of stochastic processes. Not only economists and census-takers, but engineers geologists, and all of us living in an uncertain world are being forced to accept the necessity of this new mode of thought. It will inevitably influence the arts and Xenakis has shown himself as a leader in absorbing this influence.

JANE and W.A.O'N. WAUGH

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